Higher Order Blind Source Separation Using the Cyclostationarity Property of the Signals

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Abstract

Over the last decade, a great number of higher order (HO) blind source separation methods have been developed, aiming at separating several statistically independent sources without any a priori information about the latter. Recently, a performance analysis of some of these methods has shown that the latter perform very well in a great number of situations encountered in radiocommunications contexts but fail in separating several Gaussian sources. Besides, their performance degrades in the presence of a background noise which is either spatially correlated or non Gaussian, situation encountered in the HF bandwidth. In this context, the purpose of this paper is to present and to analyse the performance of a new family of HO blind source separation methods exploiting the potential cyclostationarity of the received sources to overcome most of the limitations of the classical HO blind source separation methods while keeping their good properties.

1. Introduction

Over the last decade, a great number of higher order (HO) blind methods have been developed, aiming at separating several statistically independent sources without any a priori information about the latter[1-2]. Recently [3-4], a performance analysis of some of these methods has shown that the latter perform very well in a great number of situations encountered in radiocommunications contexts but fail in separating several Gaussian sources or need a long observation time to separate quasi-Gaussian sources such as multi-carriers waveforms. Moreover, the performance of these methods degrades in the presence of a background noise which is either spatially correlated or non Gaussian, situation encountered in particular in the HF bandwidth. A way to overcome most of these limitations of HO blind methods is to exploit other properties of the sources than their non Gaussian character such as, for example, their cyclostationarity property, for digitally modulated sources [5].

Since a few years, there is an increasing number of papers dealing with the exploitation of the cyclostationarity property of the signals in blind array filtering contexts [6-8]. However, most of these papers exploit only the information contained in the second order statistics of the observed data [6-7], which prevents these methods from blindly separating sources having the same cyclostationarity parameters.

In this context, the purpose of this paper is to present and to analyse the performance of a new family of HO blind source separation methods exploiting the potential cyclostationarity property of the received sources to overcome most of the main limitations of the classical HO blind source separators in radiocommunications environments, while keeping their good properties. This new family of methods corresponds to cyclic versions of the so-called JADE method presented in [1].

2. Hypothesis and Problem Formulation

Let us consider an array of N Narrow-Band (NB) sensors and let us call $x(t)$ the vector of the complex envelopes of the signals at the output of the sensors. Each sensor is assumed to receive a noisy instantaneous mixture of P statistically independent NB sources. Under these assumptions, the observation vector $x(t)$ can be written as

$$x(t) = \sum_{i=1}^{P} m_i(t)\, a_i + b(t) \triangleq A\; m(t) + b(t) \quad (2.1)$$

where $b(t)$ is the noise vector, assumed stationary and spatially white, $m(t)$ is the vector which components $m_i(t)$ are the complex amplitudes of the sources, assumed statistically independent and cyclostationary, A is the (N x P) full rank matrix of the steering vectors a_i.

The second order statistics of the data are described by the two Poly-Periodic (PP) correlation matrices $R_{x\varepsilon}(t,\tau)$, $\varepsilon = \pm 1$, defined by

$$R_{x\varepsilon}(t, \tau) \triangleq \mathrm{E}[x(t + \tau/2)x(t - \tau/2)^{\varepsilon\dagger}] \quad (2.2)$$

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where $x^1 \triangleq x$, $x^{-1} \triangleq x^*$ and * means complex conjugation. The cyclic correlation matrix of the vector $x(t)$ associated to the cyclic frequency α and to a given value of ε is defined by

$$R^{\alpha}_{x\varepsilon}(\tau) \triangleq \langle R_{x\varepsilon}(t, \tau)\, e^{-j2\pi\alpha t} \rangle \quad (2.3)$$

where $\langle C(t) \rangle$ corresponds to the temporal mean, over an infinite observation time, of the matrix $C(t)$. Using (2.1) into (2.3) we obtain, under the previous assumptions,

$$R^{\alpha}_{x\varepsilon}(\tau) = \sum_{i=1}^{P} \pi^{\alpha}_{i\varepsilon}(\tau)\, a_i\, a_i^{\varepsilon\dagger} + R^{\alpha}_{b\varepsilon}(\tau) \quad (2.4)$$

where $R^{\alpha}_{b\varepsilon}(\tau)$ and $\pi^{\alpha}_{i\varepsilon}(\tau)$ are the cyclic correlation matrix of the vector $b(t)$ and the cyclic correlation function of $m_i(t)$ associated to α and ε respectively. Note that $R^{\alpha}_{b\varepsilon}(\tau) = 0$ if $\varepsilon = -1$ or $\alpha \neq 0$.

In a same manner, the fourth-order statistics of the data are described by the three PP quadricovariance matrices $Q_{x\zeta}(t, \tau_1, \tau_2, \tau_3)$, where $\zeta \triangleq (\zeta_1, \zeta_2)$ and $(\zeta_1, \zeta_2) = (1, 1), (-1, 1)$ or $(-1, -1)$, defined by

$$Q_{x\zeta}(t,\tau_1,\tau_2,\tau_3) \triangleq \{\mathrm{Cum}[x_i(t)x_j(t-\tau_1)^{\zeta_1}x_k(t-\tau_2)^{\zeta_2}x_l(t-\tau_3)]\} \quad (2.5)$$

and the cyclic quadricovariance matrix of $x(t)$ associated to the cyclic frequency β and to the indice ζ is defined by

$$Q^{\beta}_{x\zeta}(\tau_1, \tau_2, \tau_3) \triangleq \langle Q_{x\zeta}(t, \tau_1, \tau_2, \tau_3)\, e^{-j2\pi\beta t} \rangle \quad (2.6)$$

Using (2.1) into (2.6), we obtain

$$Q_{x\zeta}{}^{\beta}(\tau_1,\tau_2,\tau_3) = \quad (2.7)$$
$$\sum_{i=1}^{P} c_{i\zeta}{}^{\beta}(\tau_1,\tau_2,\tau_3)\, [a_i \otimes a_i^{\zeta_1}][a_i^{\zeta_2} \otimes a_i]^T + Q^{\beta}_{b\zeta}(\tau_1,\tau_2,\tau_3)$$

where $\otimes$ is the Kronecker product, $Q^{\beta}_{b\zeta}(\tau_1,\tau_2,\tau_3)$ and $c_{i\zeta}{}^{\beta}(\tau_1,\tau_2,\tau_3)$ are the cyclic quadricovariance matrix of $b(t)$ and the cyclic quadricovariance function of $m_i(t)$ associated to β and ζ respectively. Note that $Q^{\beta}_{b\zeta}(\tau_1,\tau_2,\tau_3) = 0$ if $\zeta \neq (-1, -1)$ or $\beta \neq 0$.

Although in quasi-cyclostationary contexts, the optimal filters are PP and Widely Linear [9] in the general case, the problem of blind source separation we address in this paper is to blindly implement the Linear and Time Invariant (TI) $(N \times P)$ NB source separator W which outputs the vector $y(t) = W^{\dagger}x(t)$ and which gives the best output performance, in terms of Signal to Interference plus Noise Ratio (SINR) maximization [3-4].

3. Classical indirect HO blind source separation methods

The classical indirect HO blind source separation methods, and in particular the JADE method presented in [1], exploit only the information contained in estimates of the matrices $R_{x1}{}^0(0)$, noted R_X, and $Q^0_{x\zeta}(0, 0, 0)$ for $\zeta=(-1,-1)$, noted Q_x, which is obviously only a partial information in quasi-cyclostationary contexts. These methods aim at blindly identifying the source steering vectors before the effective source separation. This *identification* stage requires a preliminary stage of *data prewhitening* which aims at orthonormalizing the source steering vectors to be able to search, during the identification stage, for a unitary matrix, simpler to handle. Then, using the source steering vectors estimates, a *spatial filtering* operation can be implemented to separate the sources. The functional scheme of indirect blind source separators is depicted at Figure 1

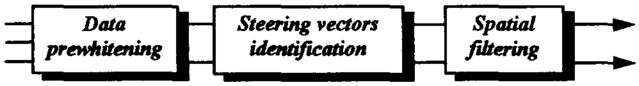

Figure 1. *Functional Scheme of indirect separators*

In the JADE method presented in [1], the prewhitening of the data is done by an estimate of the $(P \times N)$ matrix $R_s^{-1/2}$, which corresponds to the pseudo-inverse of a square root of $R_s \triangleq A\, R_m\, A^{\dagger}$, where $R_m \triangleq R_{m1}{}^0(0)$, and which is computed from the eigendecomposition of an estimate of R_x. Noting $z(t) \triangleq R_s^{-1/2}x(t)$ the observation vector whitened by $R_s^{-1/2}$, we obtain from (2.1)

$$z(t) = \sum_{i=1}^{P} m_i'(t)\; a_i' + b'(t) \triangleq A'm'(t) + b'(t) \quad (3.1)$$

where $m_i'(t)$ is the normalized complex envelope of the source i, A' is the $(P \times P)$ unitary matrix of the whitened source steering vectors a_i' and $b'(t)$ the new noise vector.

In the identification stage of the JADE method, an estimate of the unitary matrix A' is obtained by searching for a unitary matrix $\hat{A}'$ which jointly diagonalizes the P eigenmatrices of an estimate of Q_z, considered in this case as an operator on a $(P \times P)$ matrices space, associated to the P eigenvalues having the highest modulus and weighted for example by these associated eigenvalues. The matrix Q_z is defined by $Q_z \triangleq Q^0_{z\zeta}(0, 0, 0)$ for $\zeta = (-1, -1)$. Under a Gaussian noise assumption, the P eigenmatrices M_i, $(1 \leq i \leq P)$, of Q_z associated to the non zero eigenvalues have the following form [1]

$$M_i = A'\,\Lambda_i\,A'^{\dagger} \quad (3.2)$$

where the matrices Λ_i are $(P \times P)$ real-valued diagonal matrices. It is then shown in [1] that provided there is at most one Gaussian source, the matrix $\hat{A}'$ we search for corresponds, to within a diagonal and a permutation matrix, to the unitary matrix A' of the whitened source steering vectors.

Finally, from $\hat{A}'$ and the estimate of $R_s^{-1/2}$, it is possible to compute an estimate of the optimal Linear and

TI source separator which consists to implement a Spatial Matched Filter (SMF) on each source [3-4] and which is defined by an estimate of $W = R_x^{-1} (R_s^{-1/2})^{-1} \hat{A}'$.

4. THE CYCLIC JADE METHOD OF BLIND SOURCE SEPARATION

Contrary to the classical JADE method previously described, in the presence of cyclostationary sources such that at least one of these sources has the cyclic frequency α for a given value of ε, the Cyclic JADE method we propose in this paper exploits the information contained not only in an estimate of the matrix R_x, but also in an estimate of the matrix $R_{x\varepsilon}^{\alpha}(\tau)$ and in one of the three matrices $Q_{x\zeta}^{\alpha}(\tau_1, \tau_2, \tau_3)$. The parameters τ and (τ_1, τ_2, τ_3), jointly with the kind of cyclic correlation and quadricovariance matrices, are chosen such that a maximum energy is associated to the parameters (α,τ) and $(\alpha,\tau_1,\tau_2,\tau_3)$ on the corresponding second and fourth-order cyclic statistics estimate of the observed data respectively. Then, having selected all the previous parameters from the estimation of the second and fourth order statistics of the data, the philosophy of the new method is described in the following.

The ***first stage*** of the Cyclic JADE method is the prewhitening data stage of the classical JADE method.

In a ***second stage***, the dimension of the vector $z(t)$ is reduced to P_α, the number of sources having the cyclic frequency α for the given ε. This is done by projecting $z(t)$ on an estimate of the space spanned by the orthonormalized source steering vectors having this cyclic frequency α. This generates a new $(P_\alpha \times 1)$ observation vector, which is an estimate of $z^{\alpha}(t) \triangleq U_{s\varepsilon}^{\alpha\dagger} z(t)$, where the $(P \times P_\alpha)$ matrix $U_{s\varepsilon}^{\alpha}$ contains the left singular vectors associated to the non zero singular values of the matrix $R_{z\varepsilon}^{\alpha}(\tau)$, defined by (2.4) with $z(t)$ instead of $x(t)$. Note that the previous SVD gives also the $(P \times (P - P_\alpha))$ matrix $U_{b\varepsilon}^{\alpha}$ which columns span the space of the orthonormalized steering vectors associated to the sources which have not the cyclic frequency α.

In a ***third stage***, the $(P_\alpha \times P_\alpha)$ matrix A_α' of the orthonormalized steering vectors of the sources having the cyclic frequency α are estimated from one of the cyclic quadricovariance matrix $Q_{z\alpha\zeta}^{\alpha}(\tau_1, \tau_2, \tau_3)$ of $z^{\alpha}(t)$, defined by (2.6) with $z^{\alpha}(t)$ and α instead of $x(t)$ and β. Using (3.1), $Q_{z\alpha\zeta}^{\alpha}(\tau_1, \tau_2, \tau_3)$ can be written as

$$Q_{z\alpha\zeta}^{\alpha}(\tau_1,\tau_2,\tau_3) = \sum_{i=1}^{P\alpha} c_{i\zeta}^{\alpha_i}(\tau_1,\tau_2,\tau_3)[a_i' \otimes a_i'^{\zeta_1}][a_i'^{\zeta_2} \otimes a_i']^T \quad (4.1)$$

where $c_{i\zeta}^{\alpha_i}(\tau_1,\tau_2,\tau_3)$ is the cyclic quadricovariance function of $m_i'(t)$ associated to α and ζ. Note that, in this paper, we use $\zeta = (-1, 1)$ if $\varepsilon = -1$ and $\zeta = (-1, -1)$ if $\varepsilon = 1$.

Associating the matrix $v_1 v_2^T$ to the vector $v_1 \otimes v_2$, it is then easy to see that the P_α left singular matrices M_i^{α} $(1 \le i \le P_\alpha)$, which are associated to the left singular vectors of the operator $Q_{z\alpha\zeta}^{\alpha}(\tau_1,\tau_2,\tau_3)$ with non zero P_α singular values, have the following form

$$M_i^{\alpha} = A_\alpha' \Lambda_i^{\alpha} A_\alpha'^{\zeta_1 T} \quad (4.2)$$

where the matrices Λ_i^{α} are $(P_\alpha \times P_\alpha)$ complex-valued diagonal matrices. Considering now the matrices H_i^{α} defined by

$$H_i^{\alpha} \triangleq M_i^{\alpha} M_i^{\alpha\dagger} = A_\alpha' [\Lambda_i^{\alpha} \Lambda_i^{\alpha\dagger}] A_\alpha'^{\dagger} \quad (4.3)$$

we find that the Hermitian matrices H_i^{α} have the same form as the matrices M_i, defined by (3.2). Applying the results of [1], we deduce that provided there is at most one Gaussian source at the cyclic frequency α, the unitary matrix which jointly diagonalizes the matrices H_i^{α}, weighted by the associated singular values, corresponds, to within a diagonal and a permutation matrix, to the unitary matrix A_α' of the whitened source steering vectors at cyclic frequency α.

In a ***fourth stage***, the previous process (stages 2 and 3) can be reused for another cyclic frequency β to blindly estimate the orthonormalized steering vectors of sources having the cyclic frequency β. However, an alternative to this possibility is to blindly estimate the orthonormalized steering vectors of the sources which have not the cyclic frequency α by the classical JADE identification stage applied on the observation vector $z_b^{\alpha}(t) \triangleq U_{b\varepsilon}^{\alpha\dagger} z(t)$. The functional scheme of the four previously decribed stages is presented at figure 2.

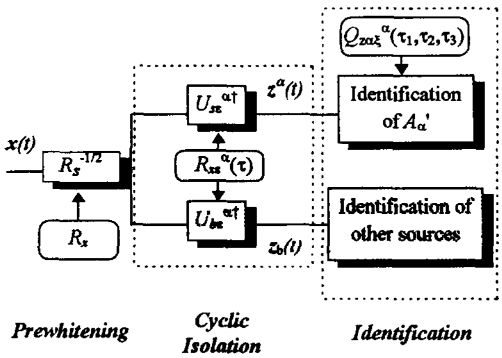

Figure 2. *Cyclic JADE functional Scheme*

Finally, although for quasi-cyclostationary observations it is advantageous to implement a PP filter from the source steering vectors estimates [10], the spatial filtering stage we implement in this paper is the same as that of the classical JADE method described previously.

Note that to obtain consistent estimates of the cyclic fourth-order cumulants of the observations, we have to take into account, in the estimation process, all the cyclic frequencies of the observations as it is shown in [11-12]. In this paper we take into account only a part of this information.

5. PERFORMANCE ILLUSTRATIONS

The figures 3 to 5 illustrate the behaviour of the JADE and Cyclic JADE methods through the variations of the output SINR of some sources for a Uniformly Spaced Circular array of 5 sensors receiving 3 sources at $\theta_1 = 50°$, $\theta_2 = 250°$ and $\theta_3 = 70°$. The output SINR of the sources 1 and 3 are presented at figures 3 and 4 respectively, when the sources 1 and 2 are BPSK modulated with the same cyclic frequencies, no carrier residue and an input SNR of 10 dB while the source 3 is a carrier which SNR is − 5 dB. For this configuration, $\alpha = 0$, $\varepsilon = -1$ and $\tau = 0$, which means that the second stage of Cyclic JADE isolates the 2 BPSK signals from the carrier. The quantity BT corresponds to the number of independent snapshots.

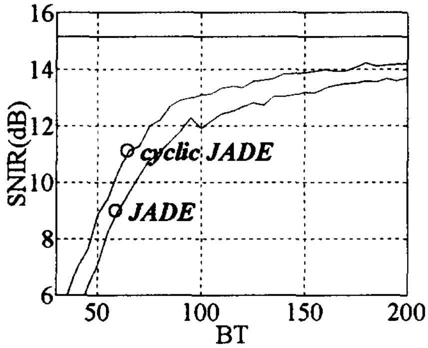

Figure 3. *SINR1 - Configuration 1*

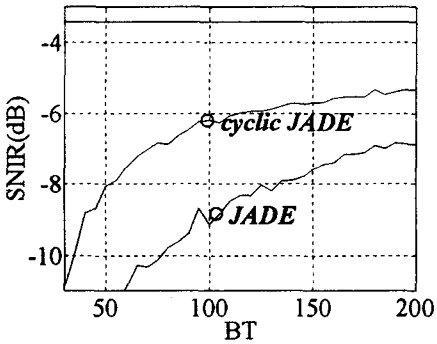

Figure 4. *SINR3 - Configuration 1*

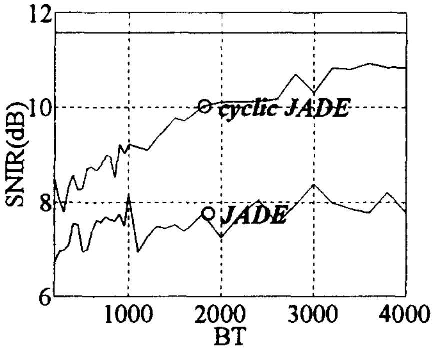

Figure 5. *SINR2 - Configuration 2*

The figure 5 depicts an other configuration and shows the output SINR of the source 2 when the source 1 is that of the previous configuration while the sources 2 and 3 are multi-carriers waveforms (quasi-Gaussian) with 32 and 40 carriers respectively. In this case, the second stage of Cyclic JADE isolates the source 2.

6. CONCLUSION

A new family of HO blind source separation methods, exploiting the potential cyclostationarity property of the sources and corresponding to cyclic extensions of the JADE method, has been presented. These methods keep the good properties of JADE method but perform better in the presence of a non Gaussian stationary noise or sources having very different input power and are able to separate Gaussian sources with different cyclic parameters.

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